Algebra:-

1. $(a+b)^2 = a^2 + b^2 + 2ab$
2. $(a-b)^2 = a^2 + b^2 - 2ab$
3. $a^2 - b^2 = (a+b)(a-b)$
4. $a^2 + b^2 = (a+b)^2 - 2ab$ or, $= (a-b)^2 + 2ab$
5. $(a-b)^2 = (a+b)^2 - 4ab$
6. $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$
7. $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 = a^3 + b^3 + 3ab(a+b)$
8. $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2 = a^3 - b^3 - 3ab(a-b)$
9. $a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)^3 - 3ab(a+b)$
10. $a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)^3 + 3ab(a-b)$
11. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
12. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$

    $\frac{1}{2}(a+b+c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$
13. $a^3 + b^3 + c^3 = 3abc, \; if \; a+b+c = 0$
14. Invertendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

    Alternendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$

    Componendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$

    Dividendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$

    Componendo and Dividendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

Surface Area and Volume:-

	CSA/LSA	TSA	VOLUME
CUBE:-	$4a^2$	$6a^2$	a^3
CUBOID:-	$2(lh+bh)$	$2(lb+bh+lh)$	lbh
CYLINDER:-	$2\pi rh$	$2\pi rh+2\pi r^2$	$\pi r^2 h$
CONE:-	πrl	$\pi rl+\pi r^2$	$\frac{1}{3}\pi r^2 h$
SPHERE:-	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
HEMISPHERE:-	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

1. The longest rod fit into a cube of side a is $\sqrt{3}a$
2. The longest rod fit into a cuboid is $\sqrt{a^2+b^2+c^2}$
3. Height of an equilateral triangle is $\frac{\sqrt{3}}{2}(side)$
4. Area of and equilateral triangle is $\frac{\sqrt{3}}{4}(side)^2$
5. Area of triangle having sides a,b and c is $\sqrt{s(s-a)(s-b)(s-c)}$, where, $s=\frac{(a+b+c)}{2}$
6. Conversion : $x\,km/hr$ = $x\times\frac{5}{18}\,m/\sec$
7. Conversion: $x\ m/\sec$ = $x\times\frac{18}{5}km/hr$
8. $1m^3=1000\ ltr=1KL$,&$10cm\times10cm\times10cm=1000cm^3=1\,ltr$
9. $1hectar=2.471acre=10,000\,m^2$
10. **K**ing, **H**enery, **D**ied, **M**other, **D**idn't, **C**ry, **M**uch

SETS:-

1. $A\Delta B=(A-B)\cup(B-A)$
2. $n(A\cup B)=n(A)+n(B)-n(A\cap B)$
3. $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$
4. Number of elements in exactly two of A,B,C $=n(A\cap B)+n(B\cap C)+n(C\cap A)-3n(A\cap B\cap C)$
5. Number of elements in exactly one of the sets A,B,C =
   $=n(A\cup B\cup C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+2n(A\cap B\cap C)$

LOGARITHM:-

1. $\log_a a=1\ ,\forall a>0\ \ and\ \ a\neq 1,$
2. $\log_a 1=0\ ,\forall a>0\ \ and\ \ a\neq 1$
3. $e^{\log_e x}=x$
4. $a^{\log_a x}=x$
5. $x^{\log_a y}=y^{\log_a x}$
6. $\log_a x=\dfrac{1}{\log_x a}$
7. $\log_{a^n} x^m=\dfrac{m}{n}\log_a x$
8. $x<y\Leftrightarrow \log_a x>\log_a y\ ,\ \forall\ 0<a<1$
9. $x<y\Leftrightarrow \log_a x<\log_a y\ ,\ \forall\ a>1$

1. $\log_a(mn)=\log_a m+\log_a n$
2. $\log_a\left(\dfrac{m}{n}\right)=\log_a m-\log_a n$
3. $\log_a m^n=n\log m$
4. $\log_y x=\dfrac{\log_k x}{\log_k y}$
5. $\log_a \sqrt[n]{x}=\dfrac{1}{n}\log_a x$
6. $\log_e x$ is also denoted as $\ln x$
7. $\ln x\ \ i.e.\ \log_e x=\dfrac{\log_{10} x}{\log_{10} e}$
8. $\log_{10} e=0.434294\ldots\approx 0.4343$
9. $\log(a\pm b)\neq\log a\pm\log b$

TRIGONOMETRY:-

1. $1^c\,(one\,radian)$ =The angle subtended at the center of a circle by an arc of length equal to the radius of the circle.
2. $\pi^c = 180^0$
3. $1^c = \dfrac{180^0}{\pi}$ 4. $1^0 = \left(\dfrac{\pi}{180}\right)^c$
4. $90^0 = 100\,grades$
5. $\dfrac{D}{90} = \dfrac{G}{100} = \dfrac{2R}{\pi}$ (Relation among degree, gradient and radian)
6. The sum of all the angles of a regular polygon is $180(n-2)$ degrees , where n is the number of sides of the polygon
7. The one angle of a regular polygon is $\dfrac{180(n-2)}{n}$ where n is the number of sides of the polygon.

Also, The angle of a regular polygon is $\left(\dfrac{2n-4}{n}\right).90\deg = \left(\dfrac{2n-4}{n}\right).100\,grades = \left(\dfrac{2n-4}{n}\right).\dfrac{\pi}{2}rad$

8. Values of Trigonometric ratios:

Trig	0^0	$30^0\left(\frac{\pi}{6}\right)$	$45^0\left(\frac{\pi}{4}\right)$	$60^0\left(\frac{\pi}{3}\right)$	$90^0\left(\frac{\pi}{2}\right)$	$180^0(\pi)$	$270^0\left(\frac{3\pi}{2}\right)$	$360^0(2\pi)$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$N.D$	0	$N.D$	0
$Cot\theta$	$N.D$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$N.D$	0	$N.D$
$Sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$N.D$	-1	$N.D$	1
$Cosec\theta$	$N.D$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$N.D$	-1	$N.D$

9. $\sin^2\theta + \cos^2\theta = 1 \quad \Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} \quad \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$
10. $\sec^2\theta - \tan^2\theta = 1 \quad \Rightarrow 1+\tan^2\theta = \sec^2\theta \quad \Rightarrow \sec^2\theta - 1 = \tan^2\theta$
11. $\cos ec^2\theta - \cot^2\theta = 1 \quad \Rightarrow 1+\cot^2\theta = \cos ec^2\theta \quad \Rightarrow \cos ec^2\theta - 1 = \cot^2\theta$

12. $Sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\quad Sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$

$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ $\quad \cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$

$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ $\quad \tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$

$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$ $\quad \cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$

$\sec\left(\frac{\pi}{2}-\theta\right)=\cos ec\theta$ $\quad \sec\left(\frac{\pi}{2}+\theta\right)=-\cos ec\theta$

$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=sec\theta$ $\quad \operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)=sec\theta$

13. $\sin(\pi-\theta)=\sin\theta$ $\quad \sin(\pi+\theta)=-\sin\theta$

$\cos(\pi-\theta)=-\cos\theta$ $\quad \cos(\pi+\theta)=-\cos\theta$

$\tan(\pi-\theta)=-\tan\theta$ $\quad \tan(\pi+\theta)=\tan\theta$

$\cot(\pi-\theta)=-\cot\theta$ $\quad \cot(\pi+\theta)=\cot\theta$

$\sec(\pi-\theta)=-\sec\theta$ $\quad \sec(\pi+\theta)=-\sec\theta$

$co\sec(\pi-\theta)=co\sec\theta$ $\quad co\sec(\pi+\theta)=-co\sec\theta$

14. $\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$ $\quad \sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$

$\cos\left(\frac{3\pi}{2}-\theta\right)=-\sin\theta$ $\quad \cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$

$\tan\left(\frac{3\pi}{2}-\theta\right)=\cot\theta$ $\quad \tan\left(\frac{3\pi}{2}+\theta\right)=-\cot\theta$

$\cot\left(\frac{3\pi}{2}-\theta\right)=\tan\theta$ $\quad \cot\left(\frac{3\pi}{2}+\theta\right)=-\tan\theta$

$\sec\left(\frac{3\pi}{2}-\theta\right)=-\cos ec\theta$ $\quad \sec\left(\frac{3\pi}{2}+\theta\right)=\cos ec\theta$

$co\sec\left(\frac{3\pi}{2}-\theta\right)=-s ec\theta$ $\quad co\sec\left(\frac{3\pi}{2}+\theta\right)=-s ec\theta$

15. $\sin(2\pi-\theta)=-\sin\theta$ $\quad \sin(2\pi+\theta)=\sin\theta$

$\cos(2\pi-\theta)=\cos\theta$ $\quad \cos(2\pi+\theta)=\cos\theta$

$\tan(2\pi-\theta)=-\tan\theta$ $\quad$ $\tan(2\pi+\theta)=\tan\theta$

$\cot(2\pi-\theta)=-\cot\theta$ $\quad$ $\cot(2\pi+\theta)=\cot\theta$

$\sec(2\pi-\theta)=\sec\theta$ $\quad$ $\sec(2\pi+\theta)=\sec\theta$

$cosec(2\pi-\theta)=-cosec\theta$ $\quad$ $cosec(2\pi+\theta)=cosec\theta$

16. $\sin 2x=2\sin x.\cos x=\dfrac{2\tan x}{1+\tan^2 x}$

17. $\cos 2x=\cos^2 x-\sin^2 x=2\cos^2 x-1=1-2\sin^2 x=\dfrac{1-\tan^2 x}{1+\tan^2 x}$

18. $\tan 2x=\dfrac{2\tan x}{1-\tan^2 x}$,

19. $\tan 3x=\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}$

20. $\sin 3x=3\sin x-4\sin^3 x$,

21. $\cos 3x=4\cos^3 x-3\cos x$,

22. $\sin(x+y)=\sin x.\cos y+\cos x.\sin y$

23. $\sin(x-y)=\sin x.\cos y-\cos x.\sin y$

24. $\cos(x+y)=\cos x.\cos y-\sin x.\sin y$

25. $\cos(x-y)=\cos x.\cos y+\sin x.\sin y$

26. $\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x.\tan y}$,

27. $\tan(x-y)=\dfrac{\tan x-\tan y}{1+\tan x.\tan y}$

28. $\cot(x+y)=\dfrac{\cot x.\cot y-1}{\cot x+\cot y}$,

29. $\cot(x-y)=\dfrac{\cot x.\cot y+1}{\cot x-\cot y}$

30. $\sin C+\sin D=2\sin\dfrac{C+D}{2}.\cos\dfrac{C-D}{2}$

31. $\sin C-\sin D=2\cos\dfrac{C+D}{2}.\sin\dfrac{C-D}{2}$

32. $\cos C+\cos D=2\cos\dfrac{C+D}{2}.\cos\dfrac{C-D}{2}$

33. $\cos C-\cos D=-2\sin\dfrac{C+D}{2}.\sin\dfrac{C-D}{2}$

34. $2\sin A.\cos B=\sin(A+B)+\sin(A-B)$

35. $2\cos A.\sin B=\sin(A+B)-\sin(A-B)$

36. $2\cos A.\cos B=\cos(A+B)+\cos(A-B)$

37. $-2\sin A.\sin B=\cos(A+B)-\cos(A-B)$

i.e. $2\sin A.\sin B=\cos(A-B)-\cos(A+B)$

38. $Sin^2A-Sin^2B=Sin(A+B).Sin(A-B)=\cos^2B-\cos^2A$

39. $Cos^2A-Sin^2B=Cos(A+B).Cos(A-B)=\cos^2B-\sin^2A$

40. $\sin(A+B+C)=\sin A cosB cosC+\cos A\sin B\cos C+\cos A\cos B\sin C-\sin A\sin B\sin C$

41. $Cos(A+B+C)=\cos A\cos B\cos C-\cos A\sin B\sin C-\sin A\cos B\sin C-\sin A\sin B\cos C$

42. $\tan(A+B+C)=\dfrac{\tan A+\tan B+\tan C-\tan A.\tan B.\tan C}{1-\tan A.\tan B-tnaB.\tan C-\tan C.\tan A}$

43. Max and min value of $a\cos\theta+b\sin\theta$, is $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

General Solutions

44. If $\sin x=0$, $then, x=n\pi, where\ n\in Z$

45. If $\cos x=0$, $then, x=(2n+1)\dfrac{\pi}{2}, where\ n\in Z$

46. If $\tan x=0$, $then, x=n\pi, where\ n\in Z$

47. If $\sin x=\sin y$, $then, x=n\pi+(-1)^n y, where\ n\in Z$

48. If $\cos x=\cos y$, $then, x=2n\pi\pm y, where\ n\in Z$

49. If $\tan x=\tan y,\ \ then, x=n\pi+y, where\ n\in Z$

50. If $\sin^2 x=\sin^2 y, \cos^2 x=\cos^2 y, \tan^2 x=\tan^2 y,\quad x=n\pi\pm y, n\in Z$

51. Sine Formula:- $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

52. Cosine Formula :- $\cos A=\dfrac{b^2+c^2-a^2}{2bc}, \cos B=\dfrac{a^2+c^2-b^2}{2ac}, \cos C=\dfrac{a^2+b^2-c^2}{2ab}$

53. Projection:-

(i) $a=b\cos C+c\cos B$,(ii) $b=c\cos A+a\cos C$,(iii $c=a\cos B+b\cos A$

54. Tangent Formula:-

(i) $\tan\dfrac{B-C}{2}=\dfrac{b-c}{b+c}\cot\dfrac{A}{2}$, (ii) $\tan\dfrac{A-B}{2}=\dfrac{a-b}{a+b}\cot\dfrac{C}{2}$ (iii)

$\tan\dfrac{C-A}{2}=\dfrac{c-a}{c+a}\cot\dfrac{B}{2}$

55. Area of the triangle:- $=\dfrac{1}{2}bc\sin A=\dfrac{1}{2}ca\sin B=\dfrac{1}{2}ab\sin C$

56. Radius of the circumcircle of the triangle ABC is

(i) $R=\frac{a}{2\sin A}=\frac{b}{2\sin B}=\frac{c}{2\sin C}$ (ii) $R=\frac{ABC}{4\Delta}$

(ii) In triangle ABC, if $8R^2=a^2+b^2+c^2$, then ABC is a right angled triangle.

57. Radius of the in circle of the triangle ABC is

(i) $r=\frac{\Delta}{s}$, *where* $s=\frac{a+b+c}{2}$ (ii) $r=4R\sin\frac{A}{2}\cdot\sin\frac{B}{2}\cdot\sin\frac{C}{2}$

Some Important Conversions: -

1. $1+\cos\theta=2\cos^2\frac{\theta}{2}$, $\left\{\because 1+2\cos^2\frac{\theta}{2}-1\right\}$
2. $1-\cos\theta=2\sin^2\frac{\theta}{2}$ $\left\{\because 1-\left(1-2\sin^2\frac{\theta}{2}\right)\right\}$
3. $1+\sin\theta=\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2$ or $\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2$, $\left\{\because \sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}$
4. $1-\sin\theta=\left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)^2$ or $\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2$, $\left\{\because \sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}$
5. $\frac{\cos x}{1+\sin x}=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}=\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}$

$=\frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}=\frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}=\frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{4}.\tan\frac{x}{2}}=\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$

6. $\frac{\sin x}{1+\cos x}=\frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}=\tan\frac{x}{2}$,
7. $\frac{\sin x}{1-\cos x}=\frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}=\cot\frac{x}{2}$

Limits and Derivatives:-

1. $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$, $n\in Q$
2. $\lim_{x\to a}k=k$
3. $\lim_{x\to 0}\frac{\sin x}{x}=1,\quad \lim_{x\to a}\frac{\sin(x-a)}{x-a}=1$
4. $\lim_{x\to 0}\frac{\tan x}{x}=1,\lim_{x\to a}\frac{\tan(x-a)}{x-a}=1$
5. $\lim_{x\to 0}\frac{e^x-1}{x}=1$
6. $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a, a>0, a\neq 0$
7. $\lim_{x\to 0}\frac{\log_e(1+x)}{x}=1$
8. Differentiability :- If $\lim_{x\to c^-}\frac{f(x)-f(c)}{x-c}=\lim_{x\to c^+}\frac{f(x)-f(c)}{x-c}$, then we can say $f(x)$ is differentiable at $x=c$.
9. $y=k$, $\frac{dy}{dx}=0$
10. $y=x^n$, $n\in R$, $\frac{dy}{dx}=nx^{n-1}$
11. $y=e^x$ $\frac{dy}{dx}=e^x$
12. $y=a^x$ $\frac{dy}{dx}=a^x\log a$
13. $y=\sqrt{x}$ $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$
14. $y=\frac{1}{x}$ $\frac{dy}{dx}=-\frac{1}{x^2}$
15. $y=\log x$ $\frac{dy}{dx}=\frac{1}{x}$
16. $y=\sin x$ $\frac{dy}{dx}=\cos x$
17. $y=\cos x$ $\frac{dy}{dx}=-\sin x$
18. $y=\tan x$ $\frac{dy}{dx}=\sec^2 x$
19. $y=\cot x$ $\frac{dy}{dx}=-\cos ec^2 x$
20. $y=\sec x$ $\frac{dy}{dx}=\sec x.\tan x$
21. $y=\cos ecx$ $\frac{dy}{dx}=-\cos ecx.\cot x$
22. $y=\sin^{-1}x$ $\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$
23. $y=\cos^{-1}x$ $\frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$
24. $y=\tan^{-1}x$ $\frac{dy}{dx}=\frac{1}{1+x^2}$
25. $y=\cot^{-1}x$ $\frac{dy}{dx}=-\frac{1}{1+x^2}$
26. $y=\sec^{-1}x$ $\frac{dy}{dx}=\frac{1}{x\sqrt{x^2-1}}$
27. $y=\cos ec^{-1}x$ $\frac{dy}{dx}=-\frac{1}{x\sqrt{x^2-1}}$

Note: Every differentiable function is continuous but vice versa is not always true.

Rules on Differentiation

1. **Product Rule:-**

If $y = U.V$, then $\frac{dy}{dx} = U.V' + V.U'$

2. **Division Rule/Quotient Rule:-**

$y = \frac{U}{V}$, $\frac{dy}{dx} = \frac{U'.V - U.V'}{V^2}$

3. **Chain Rule:-**

If $y = f[\phi(x)]$, $\frac{dy}{dx} = f'[\phi(x)].\phi'(x)$

Types of Differentiation

1. $\frac{d(\sin y)}{dx} = \cos y.\frac{dy}{dx}$

2. If $y = \left[f(x) \right]^{g(x)}$, then solve it by using logarithm both the sides.

Rolle's Theorem:-

If f is a real valued function defined in $[a,b]$, and if f is

(i) Continuous on $[a,b]$

(ii) Differentiable on (a,b)

(iii) $f(a) = f(b)$

Then there exist a $c \in (a,b)$, s:t $f'(c) = 0$

LMVT:-

If continuity and differentiable is satisfied then there exist a $c \in (a,b)$ s:t $f'(c) = \frac{f(b) - f(a)}{b - a}$

Integration:-

Integration is the reverse process of differentiation

1. $\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$ & vice versa.
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, *for* $n \neq -1$
3. $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$, *for* $n \neq 1$
4. $\int k\,dx = kx + c$
5. $\int e^x dx = e^x + c$
6. $\int e^{ax} dx = \frac{e^x}{a} + c$
7. $\int a^x dx = \frac{a^x}{\log a} + c$
8. $\int a^{bx} dx = \frac{a^{bx}}{b.(\log a)} + c$
9. $\int \frac{1}{x} dx = \log|x| + c$
10. $\int \frac{1}{ax+b} dx = \frac{\log(ax+b)}{a} + c$
11. $\int \sin x.dx = -\cos x + c$
12. $\int \cos x.dx = \sin x + c$
13. $\int \tan x.dx = \log(\sec x) + c$, *or* $-\log(\cos x) + c$
14. $\int \cot x.dx = \log(\sin x) + c$
15. $\int \sec x.dx = \log(\sec x + \tan x) + c$
16. $\int \cos ecx.dx = \log(\cos ecx - \cot x) + c$
17. $\int \sec^2 x.dx = \tan x + c$
18. $\int \cos ec^2 x.dx = -\cot x + c$
19. $\int \sec x.\tan x.dx = \sec x + c$
20. $\int \cos ecx.\cot x dx = -\cos ecx + c$
21. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c = -\cos^{-1} x + c$
22. $\int \frac{dx}{1+x^2} dx = \tan^{-1} x + c = -\cot^{-1} x + c$
23. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c = -\cos ec^{-1} x + c$
24. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$
25. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$
26. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
27. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + c$
28. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$
29. $\int \frac{dx}{\sqrt{a^2+x^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + c$
30. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + c$
31. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + c$

32. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Properties of Integration

33. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

34. $\int_a^a f(x)\,dx = 0$

35. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

36. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,$

37. $\int_0^a f(x)\,dx = \int_0^a (a-x)dx$

38. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

39. $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, \text{ for } f \text{ even} \\ 0\ , \quad \text{for } f \text{ is odd function} \end{cases}$

Note:- If $f(-x) = f(x)$, then $f(x)$ is an even function and

If $f(-x) = -f(x)$, then $f(x)$ is an odd function.

40. $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

41. $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, if\ f(2a-x) = f(x) \\ 0 \qquad\quad , if\ f(2a-x) = -f(x) \end{cases}$

Different Forms of Integration:

42. $\int e^x \left[f(x) + f'(x) \right] dx = \int e^x f(x)\,dx + c$

43. $\int e^{kx} \left\{ kf(x) + f'(x) \right\} dx = e^{kx} f(x) + c$

44. **Integration by parts:** $\int U.V\,dx = U.\int V dx - \int \left[U'.\int V.dx \right] dx$

45. Partial Fractions:

Expression of Proper fraction $\frac{P(x)}{Q(x)}$ in the sum of its partial fractions as

46. $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$, $a, b \neq 0$

47. $\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$

48. $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

49. $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$

50. $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+c}{x^2+bx+c}$, Where x^2+bx+c can't factorized further.

51. If degree of Numerator is greater or equal to denominator then,

$$\frac{Num}{Den} = Quotient + \frac{\text{Re}\,mainder}{Deno\min ator}$$

52. For, $\int \sin^m dx \; or \int \cos^m dx \qquad for \;\; m \leq 4$ use trigonometric identities.

53. $\int \sin mx.\cos nx.dx$ Using product to sum identities.

54. $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$

55. $\int f^n(x).f'(x)dx = \frac{\{f(x)\}^{n+1}}{n+1}$, $n \neq -1$

56. For, $\int \sin^m x.\cos^n x.dx$, Put , $t = \cos^n x$, $if \; m \, is \, odd$ and vice-versa.

57. For, $\int \sqrt{\frac{a-x}{a+x}} dx$ or inverse , Put $x = a\cos 2\theta$

58. $\int \sqrt{\frac{x-\alpha}{\beta-x}}$ or $\int \sqrt{(x-\alpha)(x-\beta)}$ Put $x = \alpha\cos^2\theta + \beta\sin^2\theta$

59. For $\int \frac{1}{ax^2+bx+c} dx$, Completing the squares the denominator.

60. For , $\int \frac{1}{x(x^n+1)} dx$, Multiply and divide by x^{n-1}.

61.

62. For , $\int \frac{px+q}{ax^2+bx+c}dx$, Write $px+q=\lambda\left\{\frac{d}{dx}\left(ax^2+bx+c\right)\right\}+\mu$

63. For , $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$, Write $px+q=\lambda\left\{\frac{d}{dx}\left(ax^2+bx+c\right)\right\}+\mu$

64. $\int \frac{P_{quad}}{\sqrt{Q_{quad}}}dx$, Write $P_{quad}=\lambda\left(Diff\ Q\right)+\mu\left(Q\right)+\gamma$

65. $\int \frac{\sin x+\cos x}{a\ \ term\ \ with\ \sin 2x}dx$, Put $t=\sin x-\cos x$

66. $\int \frac{1}{\sin x.f\left(\cos x\right)}dx$, Multiply and divide by $\sin x$

67. Integrals of the type, (i) $\int \frac{1}{a\sin^2 x+b\cos^2 x}dx$ (ii) $\int \frac{1}{a+b\cos^2 x}dx$ (iii)

$\int \frac{1}{a+b\sin^2 x}dx$ (iv) $\int \frac{1}{\left(a\sin x+b\cos x\right)^2}dx$ (v) $\int \frac{1}{a+b\sin^2 x+c\cos^2 x}dx$

Dividing throughout by $\cos^2 x$

68. For , (i) $\int \frac{1}{a\sin x+b\cos x}dx$ (ii) $\int \frac{1}{a+b\cos x}dx$ (iii) $\int \frac{1}{a+b\sin x}dx$ (iv)

$\int \frac{1}{a\sin x+b\cos x+c}dx$, Put $\sin x=\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\cos x=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

Or convert the denominator the sum of two terms as a single sine or cosine form.

69. For $\int \frac{a\sin x+b\cos x}{c\sin x+d\cos x}dx$ type Write

$Numerator=\lambda\left(Diff.of\ \ deno\min ator\right)+\mu\left(Deno\min ator\right)$

70. For $\int \frac{a\sin x+b\cos x+c}{p\sin x+q\cos x+r}dx$, type Write

$Numerator=\lambda\left(Diff.of\ \ deno\min ator\right)+\mu\left(Deno\min ator\right)+\nu$

71. $\int e^x\left(f\left(x\right)+f'\left(x\right)\right)dx=e^x.f\left(x\right)+c$

72. $\int e^{ax}\sin bx\,dx,\ \int e^{ax}\cos bx\,dx$, reccurring

Direct formula ,(i) $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right)+c$

(ii) $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right)+c$

73. $\int \sqrt{ax^2+bx+c}\;dx$, Completing the squares.

74. $\int (px+q)\sqrt{ax^2+bx+c}\;dx$, Write , $px+q=\lambda\frac{d}{dx}\left(ax^2+bx+c\right)+\mu$

75. Partial Fraction

76. $\int \frac{x^2+1}{x^4+\lambda x^2+1}dx$, $\int \frac{x^2-1}{x^4+\lambda x^2+1}dx$ or $\int \frac{1}{x^4+\lambda x^2+1}dx$ type, Dividing through by x^2.

77. $\int \frac{\phi(x)}{(ax+b)\sqrt{cx+d}}dx$ Put , $t^2=cx+d$

78. $\int \frac{\phi(x)}{(ax^2+bx+c)\sqrt{px+q}}dx$ Put, $t^2=px+q$

79. $\int \frac{\phi(x)}{(ax+b)\sqrt{px^2+qx+r}}dx$ Put , $ax+b=\frac{1}{t}$

80. $\int \frac{\phi(x)}{(ax^2+b)\sqrt{cx^2+d}}dx$ Put $x=\frac{1}{t}$

81. $\int \frac{f(\tan x)\,or\,1}{\sin^m x.\cos^n x}dx\; m,n\in R\; and\; m+n=even$ then divide $\sin^{m+n}x$ *or* $\cos^{m+n}x$

Inverse Trigonometry:-

1. $\sin^{-1}(\sin x) = x = \sin(\sin^{-1} x)$
2. $sin^{-1}(-x) = -\sin^{-1} x$
3. $\tan^{-1}(-x) = -\tan^{-1} x$
4. $\cos ec^{-1}(-x) = -\cos ec^{-1} x$
5. $\cos^{-1}(-x) = \pi - \cos^{-1} x$
6. $\cot^{-1}(-x) = \pi - \cot^{-1} x$
7. $\sec^{-1}(-x) = \pi - \sec^{-1} x$
8. $\sin^{-1}\left(\frac{1}{x}\right) = \cos ec^{-1} x$
9. $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$
10. $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$
11. $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
12. $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
13. $\cos ec^{-1} x + \sec^{-1} x = \frac{\pi}{2}$
14. $2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2} = \tan^{-1}\frac{2x}{1-x^2}$
15. $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}, xy < 1$
16. $\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy}, xy > -1$
17. $\tan^{-x} + \tan^{-1} y = \pi + \tan^{-1}\left(\frac{x+y}{1+xy}\right),\ for\ xy > 1, x > 0, y > 0$
18. $\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$
19. $\sin^{-1} x - \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$
20. $\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$
21. $\cos^{-1} x - \cos^{-1} y = \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)$
22. $2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$
23. $2\cos^{-1} x = \cos^{-1}\left(2x^2 - 1\right), 0 \le x \le 1$
24. $3\sin^{-1} x = \sin^{-1}\left(3x - 4x^3\right)$
25.

26. $3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right)$

27. $3\tan^{-1}x=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

For , $1-x^2$ Put $x=\sin\theta$ *or* $\cos\theta$

For, $1+x^2$, Put, $x=\tan\theta$ *or* $\cot\theta$

For, x^2-1, Put, $x=\sec\theta$ *or* $\cos ec\theta$

For, $2x^2-1$, Put, $x=\cos\theta$

For $\sqrt{a^2-x^2}$, put $x=a\sin\theta$ *or* $a\cos\theta$

For $\sqrt{a^2+x^2}$, put $x=a\tan\theta$ *or* $a\cot\theta$

For $\sqrt{x^2-a^2}$, put $x=a\sec\theta$ *or* $a\cos ec\theta$