Algebra:-

1. $(a+b)^2 = a^2 + b^2 + 2ab$
2. $(a-b)^2 = a^2 + b^2 - 2ab$
3. $a^2 - b^2 = (a+b)(a-b)$
4. $a^2 + b^2 = (a+b)^2 - 2ab$ or, $= (a-b)^2 + 2ab$
5. $(a+b)^2 = (a-b)^2 + 4ab$
6. $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$
7. $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 = a^3 + b^3 + 3ab(a+b)$
8. $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2 = a^3 - b^3 - 3ab(a-b)$
9. $a^3 + b^3 = (a+b)(a^2 - ab + b^2) = (a+b)^3 - 3ab(a+b)$
10. $a^3 - b^3 = (a-b)(a^2 + ab + b^2) = (a-b)^3 + 3ab(a-b)$
11. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
12. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ac)$

    $\frac{1}{2}(a+b+c)\left[(a-b)^2 + (b-c)^2 + (c-a)^2\right]$
13. $a^3 + b^3 + c^3 = 3abc$, *if* $a+b+c=0$
14. Invertendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}$

    Alternendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}$

    Componendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$

    Dividendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$

    Componendo and Dividendo:- $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$

Surface Area and Volume:-

	CSA/LSA	TSA	VOLUME
CUBE:-	$4a^2$	$6a^2$	a^3
CUBOID:-	$2(lh+bh)$	$2(lb+bh+lh)$	lbh
CYLINDER:-	$2\pi rh$	$2\pi rh+2\pi r^2$	$\pi r^2 h$
CONE:-	πrl	$\pi rl+\pi r^2$	$\frac{1}{3}\pi r^2 h$
SPHERE:-	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
HEMISPHERE:-	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

1. The longest rod fit into a cube of side a is $\sqrt{3}a$
2. The longest rod fit into a cuboid is $\sqrt{a^2+b^2+c^2}$
3. Height of an equilateral triangle is $\frac{\sqrt{3}}{2}(side)$
4. Area of and equilateral triangle is $\frac{\sqrt{3}}{4}(side)^2$
5. Area of triangle having sides a,b and c is $\sqrt{s(s-a)(s-b)(s-c)}$, where, $s=\frac{(a+b+c)}{2}$
6. Conversion : $x\,km/hr$ = $x\times\frac{5}{18}\,m/\sec$
7. Conversion: $x\ m/\sec$ = $x\times\frac{18}{5}km/hr$
8. $1m^3=1000\ ltr=1KL$,&$10cm\times10cm\times10cm=1000cm^3=1\ ltr$
9. $1hectar=2.471acre=10,000\,m^2$
10. **K**ing, **H**enery, **D**ied, **M**other, **D**idn't, **C**ry, **M**uch

SETS:-

1. $A\Delta B=(A-B)\cup(B-A)$
2. $n(A\cup B)=n(A)+n(B)-n(A\cap B)$
3. $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$
4. Number of elements in exactly two of A,B,C $=n(A\cap B)+n(B\cap C)+n(C\cap A)-3n(A\cap B\cap C)$
5. Number of elements in exactly one of the sets A,B,C = $=n(A\cup B\cup C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+2n(A\cap B\cap C)$

LOGARITHM:-

1. $\log_a a=1\ ,\forall a>0\ \ and\ \ a\neq 1,$
2. $\log_a 1=0\ ,\forall a>0\ \ and\ \ a\neq 1$
3. $e^{\log_e x}=x$
4. $a^{\log_a x}=x$
5. $x^{\log_a y}=y^{\log_a x}$
6. $\log_a x=\dfrac{1}{\log_x a}$
7. $\log_{a^n} x^m=\dfrac{m}{n}\log_a x$
8. $x<y\Leftrightarrow \log_a x>\log_a y\ ,\ \forall\ 0<a<1$
9. $x<y\Leftrightarrow \log_a x<\log_a y\ ,\ \forall\ a>1$

1. $\log_a(mn)=\log_a m+\log_a n$
2. $\log_a\left(\dfrac{m}{n}\right)=\log_a m-\log_a n$
3. $\log_a m^n=n\log m$
4. $\log_y x=\dfrac{\log_k x}{\log_k y}$
5. $\log_a \sqrt[n]{x}=\dfrac{1}{n}\log_a x$
6. $\log_e x$ is also denoted as $\ln x$
7. $\ln x\ \ i.e.\ \log_e x=\dfrac{\log_{10} x}{\log_{10} e}$
8. $\log_{10} e=0.434294\ldots\ 0.4343$
9. $\log(a\pm b)\neq\log a\pm\log b$

TRIGONOMETRY:-

1. $1^c\ (one\ radian)$ =The angle subtended at the center of a circle by an arc of length equal to the radius of the circle.
2. $\pi^c=180^0$
3. $1^c=\dfrac{180^0}{\pi}$
4. $1^0=\left(\dfrac{\pi}{180}\right)^c$

4. $90^0 = 100\ grades$
5. $\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$ (Relation among degree, gradient and radian)
6. The sum of all the angles of a regular polygon is $180(n-2)$ degrees , where n is the number of sides of the polygon
7. The one angle of a regular polygon is $\frac{180(n-2)}{n}$ where n is the number of sides of the polygon.

Also, The angle of a regular polygon is $\left(\frac{2n-4}{n}\right).90\deg = \left(\frac{2n-4}{n}\right).100\ grades = \left(\frac{2n-4}{n}\right).\frac{\pi}{2} rad$

8. Values of Trigonometric ratios:

Trig	0^0	$30^0\left(\frac{\pi}{6}\right)$	$45^0\left(\frac{\pi}{4}\right)$	$60^0\left(\frac{\pi}{3}\right)$	$90^0\left(\frac{\pi}{2}\right)$	$180^0(\pi)$	$270^0\left(\frac{3\pi}{2}\right)$	$360^0(2\pi)$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	$N.D$	0	$N.D$	0
$Cot\theta$	$N.D$	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$N.D$	0	$N.D$
$Sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$N.D$	-1	$N.D$	1
$Cosec\theta$	$N.D$	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$N.D$	-1	$N.D$

9. $\sin^2\theta + \cos^2\theta = 1 \quad \Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} \quad \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}$
10. $\sec^2\theta - \tan^2\theta = 1 \quad \Rightarrow 1 + \tan^2\theta = \sec^2\theta \quad \Rightarrow \sec^2\theta - 1 = \tan^2\theta$
11. $\cos ec^2\theta - \cot^2\theta = 1 \quad \Rightarrow 1 + \cot^2\theta = \cos ec^2\theta \quad \Rightarrow \cos ec^2\theta - 1 = \cot^2\theta$
12. $Sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad Sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \tan\left(\frac{\pi}{2} + \theta\right) = -\cot\theta$

$\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$ $\quad$ $\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$

$\sec\left(\frac{\pi}{2}-\theta\right)=\cos ec\theta$ $\quad$ $\sec\left(\frac{\pi}{2}+\theta\right)=-\cos ec\theta$

$\operatorname{cosec}\left(\frac{\pi}{2}-\theta\right)=sec\theta$ $\quad$ $\operatorname{cosec}\left(\frac{\pi}{2}+\theta\right)=sec\theta$

13. $\sin(\pi-\theta)=\sin\theta$ $\quad$ $\sin(\pi+\theta)=-\sin\theta$

$\cos(\pi-\theta)=-\cos\theta$ $\quad$ $\cos(\pi+\theta)=-\cos\theta$

$\tan(\pi-\theta)=-\tan\theta$ $\quad$ $\tan(\pi+\theta)=\tan\theta$

$\cot(\pi-\theta)=-\cot\theta$ $\quad$ $\cot(\pi+\theta)=\cot\theta$

$\sec(\pi-\theta)=-\sec\theta$ $\quad$ $\sec(\pi+\theta)=-\sec\theta$

$co\sec(\pi-\theta)=co\sec\theta$ $\quad$ $co\sec(\pi+\theta)=-co\sec\theta$

14. $\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$ $\quad$ $\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$

$\cos\left(\frac{3\pi}{2}-\theta\right)=-\sin\theta$ $\quad$ $\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$

$\tan\left(\frac{3\pi}{2}-\theta\right)=\cot\theta$ $\quad$ $\tan\left(\frac{3\pi}{2}+\theta\right)=-\cot\theta$

$\cot\left(\frac{3\pi}{2}-\theta\right)=\tan\theta$ $\quad$ $\cot\left(\frac{3\pi}{2}+\theta\right)=-\tan\theta$

$\sec\left(\frac{3\pi}{2}-\theta\right)=-\cos ec\theta$ $\quad$ $\sec\left(\frac{3\pi}{2}+\theta\right)=\cos ec\theta$

$co\sec\left(\frac{3\pi}{2}-\theta\right)=-\sec\theta$ $\quad$ $co\sec\left(\frac{3\pi}{2}+\theta\right)=-\sec\theta$

15. $\sin(2\pi-\theta)=-\sin\theta$ $\quad$ $\sin(2\pi+\theta)=\sin\theta$

$\cos(2\pi-\theta)=\cos\theta$ $\quad$ $\cos(2\pi+\theta)=\cos\theta$

$\tan(2\pi-\theta)=-\tan\theta$ $\quad$ $\tan(2\pi+\theta)=\tan\theta$

$\cot(2\pi-\theta)=-\cot\theta$ $\quad$ $\cot(2\pi+\theta)=\cot\theta$

$\sec(2\pi-\theta)=\sec\theta$ $\quad$ $\sec(2\pi+\theta)=\sec\theta$

$co\sec(2\pi-\theta)=-co\sec\theta$ $\quad$ $co\sec(2\pi+\theta)=co\sec\theta$

16. $\sin 2x = 2\sin x.\cos x = \dfrac{2\tan x}{1+\tan^2 x}$

17. $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

18. $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$,

19. $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$

20. $\sin 3x = 3\sin x - 4\sin^3 x$,

21. $\cos 3x = 4\cos^3 x - 3\cos x$,

22. $\sin(x+y) = \sin x.\cos y + \cos x.\sin y$

23. $\sin(x-y) = \sin x.\cos y - \cos x.\sin y$

24. $\cos(x+y) = \cos x.\cos y - \sin x.\sin y$

25. $\cos(x-y) = \cos x.\cos y + \sin x.\sin y$

26. $\tan(x+y) = \dfrac{\tan x + \tan y}{1-\tan x.\tan y}$,

27. $\tan(x-y) = \dfrac{\tan x - \tan y}{1+\tan x.\tan y}$

28. $\cot(x+y) = \dfrac{\cot x.\cot y - 1}{\cot x + \cot y}$,

29. $\cot(x-y) = \dfrac{\cot x.\cot y + 1}{\cot y - \cot x}$

30. $\sin C + \sin D = 2\sin\dfrac{C+D}{2}.\cos\dfrac{C-D}{2}$

31. $\sin C - \sin D = 2\cos\dfrac{C+D}{2}.\sin\dfrac{C-D}{2}$

32. $\cos C + \cos D = 2\cos\dfrac{C+D}{2}.\cos\dfrac{C-D}{2}$

33. $\cos C - \cos D = -2\sin\dfrac{C+D}{2}.\sin\dfrac{C-D}{2}$

34. $2\sin A.\cos B = \sin(A+B) + \sin(A-B)$

35. $2\cos A.\sin B = \sin(A+B) - \sin(A-B)$

36. $2\cos A.\cos B = \cos(A+B) + \cos(A-B)$

37. $-2\sin A.\sin B = \cos(A+B) - \cos(A-B)$

i.e. $2\sin A.\sin B = \cos(A-B) - \cos(A+B)$

38. $Sin^2 A - Sin^2 B = Sin(A+B).Sin(A-B) = \cos^2 B - \cos^2 A$

39. $Cos^2 A - Sin^2 B = Cos(A+B).Cos(A-B) = \cos^2 B - \sin^2 A$

40. $\sin(A+B+C) = \sin A cos B cos C + \cos A\sin B\cos C + \cos A\cos B\sin C - \sin A\sin B\sin C$

41. $Cos(A+B+C)=\cos A\cos B\cos C-\cos A\sin B\sin C-\sin A\cos B\sin C-\sin A\sin B\cos C$

42. $\tan(A+B+C)=\dfrac{\tan A+\tan B+\tan C-\tan A.\tan B.\tan C}{1-\tan A.\tan B-tnaB.\tan C-\tan C.\tan A}$

43. Max and min value of $a\cos\theta+b\sin\theta$, is $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

General Solutions

44. If $\sin x=0$, $then, x=n\pi, where\ n\in Z$

45. If $\cos x=0$, $then, x=(2n+1)\dfrac{\pi}{2}, where\ n\in Z$

46. If $\tan x=0$, $then, x=n\pi, where\ n\in Z$

47. If $\sin x=\sin y$, $then, x=n\pi+(-1)^n y, where\ n\in Z$

48. If $\cos x=\cos y$, $then, x=2n\pi\pm y, where\ n\in Z$

49. If $\tan x=\tan y,\ \ then, x=n\pi+y, where\ n\in Z$

50. If $\sin^2 x=\sin^2 y, \cos^2 x=\cos^2 y, \tan^2 x=\tan^2 y,\quad x=n\pi\pm y, n\in Z$

51. Sine Formula:- $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

52. Cosine Formula :- $\cos A=\dfrac{b^2+c^2-a^2}{2bc}, \cos B=\dfrac{a^2+c^2-b^2}{2ac}, \cos C=\dfrac{a^2+b^2-c^2}{2ab}$

53. Projection:-

(i) $a=b\cos C+c\cos B$,(ii) $b=c\cos A+a\cos C$,(iii $c=a\cos B+b\cos A$

54. Tangent Formula:-

(i) $\tan\dfrac{B-C}{2}=\dfrac{b-c}{b+c}\cot\dfrac{A}{2}$, (ii) $\tan\dfrac{A-B}{2}=\dfrac{a-b}{a+b}\cot\dfrac{C}{2}$ (iii) $\tan\dfrac{C-A}{2}=\dfrac{c-a}{c+a}\cot\dfrac{B}{2}$

55. Area of the triangle:- $=\dfrac{1}{2}bc\sin A=\dfrac{1}{2}ca\sin B=\dfrac{1}{2}ab\sin C$

56. Radius of the circumcircle of the triangle ABC is

(i) $R=\dfrac{a}{2\sin A}=\dfrac{b}{2\sin B}=\dfrac{c}{2\sin C}$ (ii) $R=\dfrac{ABC}{4\Delta}$

(ii) In triangle ABC , if $8R^2=a^2+b^2+c^2$,then ABC is a right angled triangle.

57. Radius of the in circle of the triangle ABC is

(i) $r=\dfrac{\Delta}{s}$, $where\ s=\dfrac{a+b+c}{2}$ (ii) $r=4R\sin\dfrac{A}{2}\cdot\sin\dfrac{B}{2}\cdot\sin\dfrac{C}{2}$

Some Important Rules: -

1. $1+\cos\theta = 2\cos^2\frac{\theta}{2}, \qquad \left\{\because 1+2\cos^2\frac{\theta}{2}-1\right\}$

2. $1-\cos\theta = 2\sin^2\frac{\theta}{2} \qquad \left\{\because 1-\left(1-2\sin^2\frac{\theta}{2}\right)\right\}$

3. $1+\sin\theta = \left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2$ or $\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)^2, \qquad \left\{\because \sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}$

4. $1-\sin\theta = \left(\sin\frac{\theta}{2}-\cos\frac{\theta}{2}\right)^2$ or $\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)^2, \qquad \left\{\because \sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right\}$

5. $$\frac{\cos x}{1+\sin x} = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2} = \frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}$$

$$= \frac{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)} = \frac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}} = \frac{\tan\frac{\pi}{4}-\tan\frac{x}{2}}{1+\tan\frac{\pi}{4}.\tan\frac{x}{2}} = \tan\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

6. $$\frac{\sin x}{1+\cos x} = \frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\cos^2\frac{x}{2}} = \tan\frac{x}{2},$$

7. $$\frac{\sin x}{1-\cos x} = \frac{2\sin\frac{x}{2}.\cos\frac{x}{2}}{2\sin^2\frac{x}{2}} = \cot\frac{x}{2}$$

Complex numbers:-

1. $i^2 = -1$
2. $|z| = \sqrt{a^2 + b^2}$
3. $|z|^2 = z \cdot \overline{z}$
4. $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$
5. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
6. $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

1. If $z = a + ib$, then $\overline{z} = a - ib$
2. $\tan\theta = \left|\frac{b}{a}\right|$, θ in its respective quadrant.
3. $\overline{(z_1 \pm z_2)} = \overline{z}_1 \pm \overline{z}_2$
4. $\overline{(z_1 \cdot z_2)} = \overline{z}_1 \cdot \overline{z}_2$
5. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}$
6. $(\cos\theta + i\sin\theta)^{-1} = \cos\theta - i\sin\theta$

Permutations and Combinations:-

1. **Fundamental principle of multiplication:-** When two jobs are completed in m and n ways , then the two jobs in succession can be completed in $m \times n$ ways.
2. **Fundamental principle of addition:-** If there are two jobs such that they can be performed independently in m and n ways respectively, then either of the two jobs can be performed in $(m+n)$ ways.
3. If n and r are positive integers such that $1 \le r \le n$, then the number of all permutations of n distinct objects taken r at a time is denoted by the symbol, $P(n,r)$ or ${}^nP_r$.
4. The number of n different things taken r at a time when each may be repeated any number of times in each arrangement is n^r.
5. The number of permutations of n distinct objects , taken all at a time is ${}^nP_n$ *i.e.* $n!$
6. The number of all permutations of n different objects taken r at a time , when a particular object is to be always included in each arrangement, is = $r \cdot {}^{n-1}P_{r-1}$
7. The number of all permutations of n different objects taken r at a time , when a particular object is never taken in each arrangement is = ${}^{n-1}P_r$
8. The number of all permutations of n different objects taken r at a time, in which two specified objects always occur together is = $2!(r-1)^{n-2}P_{r-2}$

9. The number of permutations of n objects in which p objects are alike of one kind , q objects are alike of another kind, r objects are alike of other kind and so on is = $\frac{n!}{p!\times q!\times r!\times \cdots}$
10. When the number of certain things not to occur together is , two. (e.g.- If the best and worst paper not occur together.) :- Total number of permutations minus number of ways the two papers are always together.
11. When the number of things not occurring together is more than two:- This can be done by arranging even and odd places separately.
12. The number of **circular permutations** of n different object is $(n-1)!$
13. The number of **circular** permutations of n different objects in which **two** particular objects occur **together** is $(n-2)!$
14. If n persons sit **around a table** so that all shall **not** have the **same neighbors** in any two arrangements is $\frac{(n-1)!}{2}$, Since clockwise and anticlockwise direction does matter.
15. The number of ways in which n different **beads** can be arranged to form a **necklace** is $\frac{(n-1)!}{2}$
16. (i) ${}^nC_r = {}^nC_{n-r}$ (ii) ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(iii) $\frac{n}{r}\cdot {}^{n-1}C_{r-1} = {}^nC_r$ (iv) ${}^nC_x = {}^nC_y \Rightarrow x = y \text{ , } or \text{ , } x+y=n$

(v) $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$ (vi) $\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}$

Binomial Theorem:-

1. General Expansion of Binomial theorem:-

$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$

$\Rightarrow (a+b)^n = {}^nC_0 a^{n-0} b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 \cdots\cdots\cdots + {}^nC_n a^{n-n} b^n$

(i) It has $(n+1)$ terms

(ii) The sum of indices of a and b in each term is n.

(iii) The coefficient of terms equidistant from the beginning and the end are equal

2. $(a+b)^n + (a-b)^n = 2\left[{}^nC_0 a^{n-0} b^0 + {}^nC_2 a^{n-2} b^2 + {}^nC_4 a^{n-4} b^4 \cdots\cdots\right]$
3. $(a+b)^n - (a-b)^n = 2\left[{}^nC_1 a^{n-1} b^1 + {}^nC_3 a^{n-3} b^3 + {}^nC_5 a^{n-5} b^5 \cdots\cdots\right]$
4. General Term of Binomial:- $T_{r+1} = {}^nC_r a^{n-r} b^r$
5. Middle Term of Binomial:- In the expansion of $(a+b)^n$, there are $n+1$ number of terms.
6. So, If $\underline{n = even}$, the only middle term is $\left(\frac{n}{2}+1\right)^{th}$ term.
7. If $\underline{n = odd}$, the two middle terms are $\left(\frac{n+1}{2}\right)^{th}$ and the next *i.e.* $\left(\frac{n+1}{2}+1\right)^{th}$
8. In $(a+b)^n$, the r^{th} term from the end is the $(n-r+2)^{th}$ term from the beginning.
9. $(1+a)^n = 1 + na + \frac{n(n-1)a^2}{2!} + \frac{n(n-1)(n-2)a^3}{3!} \cdots\cdots\infty$

   This expansion is valid only for $-1 < a < 1$

<u>Sequence and Series:-</u>

1. A sequence is a function whose domain is the set of all natural numbers or subset of it.
2. The sum of the terms of the sequence is called as a series.
3. Those sequence whose terms follow certain patterns are called progressions.

AP:-

(i) $a_n = a + (n-1)d$

(ii) If an AP consists of n terms,then r^{th} term from the end is the $(n-r+1)^{th}$ term from the beginning.

(iii) $S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}[a + a_n]$

(iv) $a_n = S_n - S_{n-1}$

(v) In an AP the sum of the terms equidistant from the beginning and end is always same and equal to the sum of first and last term. *i.e.* $a_k + a_{n-(k-1)} = a_1 + a_n$

(vi) If a constant is added to or subtracted from each term of an AP , then the resulting sequence is also an AP with the same common difference.

(vii) If each term of a given AP is multiplied or divided by a non-zero constant k , then the resulting sequence is also an AP with common difference kd or $\frac{k}{d}$.

(viii) The single arithmetic mean between a and b is $A.M. = \frac{a+b}{2}$

(ix) Three numbers a, b, c are in AP then $b = \frac{a+c}{2}$, and b is called the arithmetic mean of a *and* c , *i.e* $2b = a + c$.

(x) If the terms of an AP are chosen at regular intervals, then they form an AP.

(xi) If n arithmetic means are inserted between a and b , then $d = \frac{b-a}{n+1}$. Also sum of all A.M is $n\left(\frac{a+b}{2}\right)$.

(xii) The sum of n arithmetic mean between two numbers is n times the single arithmetic mean between them.

Selection of terms in A.P.

If AP consists of 3 terms:- $a-d,\ a,\ a+d$

If AP consists of 4 terms:- $a-3d,\ a-d,\ a+d, a+3d$

GP:-(i) $a_n = ar^{n-1}$ and m^{th} term from the end is ar^{n-m}

(ii) $S_n = \dfrac{a\left(r^n - 1\right)}{r-1}$ OR $S_n = \dfrac{a\left(1-r^n\right)}{1-r}, r<1$, Hence, $S_\infty = \dfrac{a}{1-r}$, for $-1<r<1$

(iii) Selection of Terms:

If GP consists of 3 terms :- $\dfrac{a}{r}, a, ar$

If GP consists of 4 terms :- $\dfrac{a}{r^3}, \dfrac{a}{r}, ar, ar^3$

(iv) The geometric mean between a and b is $\sqrt{ab}$.

(v) If a and b are of opposite sign, then the geometric mean does not exist.

(vi) If n geometric means are inserted between two quantities, then the product of n geometric means is the n^{th} power of the single geometric mean between the two quantities.

(vii) For any two number $A.M. > G.M$

(viii) a and b are the two roots of the quadratic equation $x^2 - 2Ax + G^2 = 0$

i.e. $x^2 - (a+b)x + ab = 0$

(ix)If $A and\ G$ are the arithmetic and geometric mean of two numbers a and b, then

$a, b = A \pm \sqrt{A^2 - G^2}$

4. Special Series:-

For every series $S_n = \sum_{n=1}^{n} a_n$

(i) $\sum 1 = n$

(ii) $\sum n = 1 \cdot 2 \cdot 3 \cdot 4 \cdots\cdots n = \dfrac{n(n+1)}{2}$

(iii) $\sum n^2 = 1^2 + 2^2 + 3^2 \cdots\cdots n^2 = \dfrac{n(n+1)(2n+1)}{6}$

(iv) $\sum n^3 = 1^3 + 2^3 + 3^3 \cdots\cdots n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

(v) $\sum n^4 = 1^4 + 2^4 + 3^4 \cdots\cdots n^4 = \dfrac{n(n+1)(2n+1)\left(3n^2+3n+1\right)}{30}$

Coordinate:-

1. **Distance Formula:-** If $A(x_1, y_1)$, $B(x_2, y_2)$, then $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. **Section Formula:-** If $A(x_1, y_1)$, $B(x_2, y_2)$,

(i) **Internal division:-** If C divides AB in the ratio m:n internally , then

$C\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

(ii) **Mid Point formula:-** $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

(iii) **External division:-** If C divides AB in the ratio m:n externally, then

$C\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

3. **Area of the triangle:-** $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$
4. **Centroid:-** $C\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
5. **In-Centre:-** $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$
6. **Slope of a line :-** (i) $m = \tan\theta$, (ii) $m = \frac{y_2 - y_1}{x_2 - x_1}$

(iii) $m_1 = m_2$, if two lines are parallel

(iv) $m_1 \times m_2 = -1$, if two lines are perpendicular. Or ($m_1 = \frac{-1}{m_2}$)

(Also two lines are perpendicular if $a_1a_2 + b_1b_2 = 0$)

(v)If angle between two lines is θ , then $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

(vi) The slope of the line $ax + by + cz = 0$ is $-\frac{a}{b}$.

(Note:- Angle θ is always measured anticlockwise from x-axis.)

7. **Equations of lines:-**

(i) Slope-y Intercept Form:- $y = mx + c$

(ii) Slope-x intercept Form:- $y = m(x-d)$

(iii) Intercept Form:- $\frac{x}{a}+\frac{y}{b}=1$

(iv) Slope-Point Form:- $y-y_1 = m(x-x_1)$

(v) Two point form:- $y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$

(vi) Normal Form:- $x\cos\alpha + y\sin\alpha = p$

8. Distance of a point from line: $d = \left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

9. Distance between two parallel lines:- $d = \left|\frac{c_2-c_1}{\sqrt{a^2+b^2}}\right|$

10. The length of the perpendicular from the origin to the line $ax+by+cz=0$ is $\frac{|c|}{\sqrt{a^2+b^2}}$.

Conic Section:-

1. Equation of the circle:-

(i) $(x-a)^2+(y-b)^2=r^2$, Centre=(a,b) , radius=r

(ii) $x^2+y^2+2gx+2fy+c=0$, Centre= $(-g,-f)$, radius= $\sqrt{g^2+f^2-c}$

$\therefore$ $ax^2+ay^2+2gx+2fy+c=0$ is equation of circle.

(iii) If (x_1,y_1) *and* (x_2,y_2) are the end points of the diameter of the circle then the equation of the circle is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.

2. Equation of parabola:- $y^2=4ax$

(i) Vertex = $(0,0)$

(ii) Axis along $x-axis$

(iii) Focus at $F(a,0)$

(iv) Equation of directrix, $x=-a$

(v) Length of latus rectum = $4a$

(vi) In parabola $e=1$, (e =eccentricity)

3. Equation of ellipse:-

(i) Coordinate of centre=$(0,0)$

(ii) Major axis along $x-axis$

(iii) Coordinate of Vertices=$(\pm a,0)$

(iv) Coordinate of foci=$(\pm ae,0)$, Here $e<1$

(v) Length of major axis=$2a$

(vi) Length of minor axis=$2b$

(vii) Equations of directrices=$x=\pm\frac{a}{e}$, Here $e<1$

(viii) Length of latus rectum=$\frac{2b^2}{a}$

(ix) Relation among a,b,c are $a^2=b^2+c^2$, *i.e.* $e=\sqrt{1-\frac{b^2}{a^2}}$

(x) If P is a point on the ellipse, then $PF_1+PF_2=2a$.

(xi) Equations of ellipse are $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a^2>b^2$ and $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1,\ a^2>b^2$

4. Equations of Hyperbola:-

(i) Coordinate of centre=$(0,0)$

(ii) Transverse axis along $x-axis$

(iii) Coordinate of Vertices=$(\pm a,0)$

(iv) Coordinate of foci=$(\pm ae,0)$, Here $e>1$

(v) Length of transverse axis=$2a$

(vi) Length of conjugate axis=$2b$

(vii) Equations of directrices=$x=\pm\frac{a}{e}$, Here $e>1$

(viii) Length of latus rectum=$\frac{2b^2}{a}$

(ix) Relation among a,b,c are $c^2=a^2+b^2$, *i.e.* $e=\sqrt{1+\frac{b^2}{a^2}}$

(x) Equations of hyperbola are $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

(xi) If the centre at (h,k) and the axes are parallel to the coordinate axes , then the equation of hyperbola is $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1.$

Limits and Derivatives:-

1. $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$, $n\in Q$
2. $\lim_{x\to a}k=k$
3. $\lim_{x\to 0}\frac{\sin x}{x}=1,\quad \lim_{x\to a}\frac{\sin(x-a)}{x-a}=1$
4. $\lim_{x\to 0}\frac{\tan x}{x}=1,\lim_{x\to a}\frac{\tan(x-a)}{x-a}=1$
5. $\lim_{x\to 0}\frac{e^x-1}{x}=1$
6. $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a, a>0, a\neq 0$
7. $\lim_{x\to 0}\frac{\log_e(1+x)}{x}=1$
8. Differentiability :- If $\lim_{x\to c^-}\frac{f(x)-f(c)}{x-c}=\lim_{x\to c^+}\frac{f(x)-f(c)}{x-c}$, then we can say $f(x)$ is differentiable at $x=c$.
9. $y=k$, $\frac{dy}{dx}=0$
10. $y=x^n$, $n\in R$, $\frac{dy}{dx}=nx^{n-1}$
11. $y=e^x$ $\frac{dy}{dx}=e^x$
12. $y=a^x$ $\frac{dy}{dx}=a^x\log a$
13. $y=\sqrt{x}$ $\frac{dy}{dx}=\frac{1}{2\sqrt{x}}$
14. $y=\frac{1}{x}$ $\frac{dy}{dx}=-\frac{1}{x^2}$
15. $y=\log x$ $\frac{dy}{dx}=\frac{1}{x}$
16. $y=\sin x$ $\frac{dy}{dx}=\cos x$
17. $y=\cos x$ $\frac{dy}{dx}=-\sin x$
18. $y=\tan x$ $\frac{dy}{dx}=\sec^2 x$

19. $y=\cot x \quad \frac{dy}{dx}=-\cos ec^2 x$

20. $y=\sec x \quad \frac{dy}{dx}=\sec x.\tan x$

21. $y=\cos ecx \ \frac{dy}{dx}=-\cos ecx.\cot x$

22. $y=\sin^{-1}x \ \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$

23. $y=\cos^{-1}x \ \frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}}$

24. $y=\tan^{-1}x \ \frac{dy}{dx}=\frac{1}{1+x^2}$

25. $y=\cot^{-1}x \ \frac{dy}{dx}=-\frac{1}{1+x^2}$

26. $y=\sec^{-1}x \ \frac{dy}{dx}=\frac{1}{x\sqrt{x^2-1}}$

27. $y=\cos ec^{-1}x \ \frac{dy}{dx}=-\frac{1}{x\sqrt{x^2-1}}$

Note: Every differentiable function is continuous but vice versa is not always true.

Rules on Differentiation

1. **Product Rule:-** If $y=U.V$, then $\frac{dy}{dx}=U.V'+V.U'$
2. **Division Rule/Quotient Rule:** $y=\frac{U}{V}, \ \frac{dy}{dx}=\frac{U'.V-U.V'}{V^2}$
3. **Chain Rule:** If $y=f[\phi(x)]$, $\frac{dy}{dx}=f'[\phi(x)].\phi'(x)$

Types of Differentiation

1. $\frac{d(\sin y)}{dx}=\cos y.\frac{dy}{dx}$
2. If $y=\left[f(x)\right]^{g(x)}$, then solve it by using logarithm both the sides.

Rolle's Theorem:-

If f is a real valued function defined in $[a,b]$, and if f is

(i) Continuous on $[a,b]$

(ii) Differentiable on (a,b)

(iii) $f(a)=f(b)$

Then there exist a $c\in(a,b)$, s:t $f'(c)=0$

LMVT:-

If continuity and differentiable is satisfied then there exist a $c\in(a,b)$ s:t $f'(c)=\frac{f(b)-f(a)}{b-a}$

Integration:-

Integration is the reverse process of differentiation

1. $\frac{d}{dx}\left[\int f(x)\,dx\right] = f(x)$ & vice versa.
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, *for* $n \neq -1$
3. $\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$, *for* $n \neq 1$
4. $\int k\,dx = kx + c$
5. $\int e^x dx = e^x + c$
6. $\int e^{ax} dx = \frac{e^x}{a} + c$
7. $\int a^x dx = \frac{a^x}{\log a} + c$
8. $\int a^{bx} dx = \frac{a^{bx}}{b.(\log a)} + c$
9. $\int \frac{1}{x} dx = \log|x| + c$
10. $\int \frac{1}{ax+b} dx = \frac{\log(ax+b)}{a} + c$
11. $\int \sin x.dx = -\cos x + c$
12. $\int \cos x.dx = \sin x + c$
13. $\int \tan x.dx = \log(\sec x) + c$, *or* $-\log(\cos x) + c$
14. $\int \cot x.dx = \log(\sin x) + c$
15. $\int \sec x.dx = \log(\sec x + \tan x) + c$
16. $\int \cos ecx.dx = \log(\cos ecx - \cot x) + c$
17. $\int \sec^2 x.dx = \tan x + c$
18. $\int \cos ec^2 x.dx = -\cot x + c$
19. $\int \sec x.\tan x.dx = \sec x + c$
20. $\int \cos ecx.\cot x dx = -\cos ecx + c$
21. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c = -\cos^{-1} x + c$
22. $\int \frac{dx}{1+x^2} dx = \tan^{-1} x + c = -\cot^{-1} x + c$
23. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c = -\cos ec^{-1} x + c$
24. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$
25. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$
26. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
27. $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x + \sqrt{x^2-a^2}\right| + c$
28. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$
29. $\int \frac{dx}{\sqrt{a^2+x^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + c$
30. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + c$
31. $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + c$
32. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Properties of Integration

33. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

34. $\int_a^a f(x)\,dx = 0$

35. $\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

36. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx,$

37. $\int_0^a f(x)\,dx = \int_0^a (a-x)dx$

38. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

39. $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, \text{ for } f \text{ even} \\ 0\ , \quad \text{for } f \text{ is odd function} \end{cases}$

Note:- If $f(-x) = f(x)$, then $f(x)$ is an even function and

If $f(-x) = -f(x)$, then $f(x)$ is an odd function.

40. $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

41. $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx, \text{if } f(2a-x) = f(x) \\ 0 \qquad\quad \text{, if } f(2a-x) = -f(x) \end{cases}$

Different Forms of Integration:

42. $\int e^x \left[f(x) + f'(x) \right] dx = \int e^x f(x)\,dx + c$

43. $\int e^{kx} \left\{ kf(x) + f'(x) \right\} dx = e^{kx} f(x) + c$

44. **Integration by parts:** $\int U.V\,dx = U.\int V dx - \int \left[U'.\int V.dx \right] dx$

45. **Partial Fractions:**

Expression of Proper fraction $\frac{P(x)}{Q(x)}$ in the sum of its partial fractions as

46. $\frac{px+q}{(x-a)(x-b)}=\frac{A}{x-a}+\frac{B}{x-b}$, $a,b\neq 0$

47. $\frac{px+q}{(x-a)^2}=\frac{A}{x-a}+\frac{B}{(x-a)^2}$

48. $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}=\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$

49. $\frac{px^2+qx+r}{(x-a)^2(x-b)}=\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$

50. $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}=\frac{A}{x-a}+\frac{Bx+c}{x^2+bx+c}$, Where x^2+bx+c can't factorized further.

51. If degree of Numerator is greater or equal to denominator then,

$$\frac{Num}{Den}=Quotient+\frac{\mathrm{Re}\,mainder}{Deno\,\mathrm{min}\,ator}$$

52. For, $\int \sin^m dx \; or \int \cos^m dx \qquad for \;\; m\leq 4$ use trigonometric identities.

53. $\int \sin mx.\cos nx.dx$ Using product to sum identities.

54. $\int \frac{f'(x)}{f(x)}dx=\log|f(x)|+c$

55. $\int f^n(x).f'(x)dx=\frac{\{f(x)\}^{n+1}}{n+1}$, $n\neq -1$

56. For, $\int \sin^m x.\cos^n x.dx$, Put , $t=\cos^n x$, $if\; m\,is\,odd$ and vice-versa.

57. For, $\int\sqrt{\frac{a-x}{a+x}}dx$ or inverse , Put $x=a\cos 2\theta$

58. $\int\sqrt{\frac{x-\alpha}{\beta-x}}$ or $\int\sqrt{(x-\alpha)(x-\beta)}$ Put $x=\alpha\cos^2\theta+\beta\sin^2\theta$

59. For $\int\frac{1}{ax^2+bx+c}dx$, Completing the squares the denominator.

60. For , $\int\frac{1}{x(x^n+1)}dx$, Multiply and divide by x^{n-1}.

61. For , $\int\frac{px+q}{ax^2+bx+c}dx$, Write $px+q=\lambda\left\{\frac{d}{dx}(ax^2+bx+c)\right\}+\mu$

62. For , $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$, Write $px+q=\lambda\left\{\frac{d}{dx}\left(ax^2+bx+c\right)\right\}+\mu$

63. $\int \frac{P_{quad}}{\sqrt{Q_{quad}}}dx$, Write $P_{quad}=\lambda\left(Diff\ Q\right)+\mu\left(Q\right)+\gamma$

64. $\int \frac{\sin x+\cos x}{a\ \ term\ \ with\ \ \sin 2x}dx$, Put $t=\sin x-\cos x$

65. $\int \frac{1}{\sin x.f\left(\cos x\right)}dx$, Multiply and divide by $\sin x$

66. Integrals of the type, (i) $\int \frac{1}{a\sin^2 x+b\cos^2 x}dx$ (ii) $\int \frac{1}{a+b\cos^2 x}dx$ (iii)

$\int \frac{1}{a+b\sin^2 x}dx$ (iv) $\int \frac{1}{\left(a\sin x+b\cos x\right)^2}dx$ (v) $\int \frac{1}{a+b\sin^2 x+c\cos^2 x}dx$

Dividing throughout by $\cos^2 x$

67. For , (i) $\int \frac{1}{a\sin x+b\cos x}dx$ (ii) $\int \frac{1}{a+b\cos x}dx$ (iii) $\int \frac{1}{a+b\sin x}dx$ (iv)

$\int \frac{1}{a\sin x+b\cos x+c}dx$, Put $\sin x=\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\cos x=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

Or convert the denominator the sum of two terms as a single sine or cosine form.

68. For $\int \frac{a\sin x+b\cos x}{c\sin x+d\cos x}dx$ type Write

$Numerator=\lambda\left(Diff.of\ \ deno\min ator\right)+\mu\left(Deno\min ator\right)$

69. For $\int \frac{a\sin x+b\cos x+c}{p\sin x+q\cos x+r}dx$, type Write

$Numerator=\lambda\left(Diff.of\ \ deno\min ator\right)+\mu\left(Deno\min ator\right)+\nu$

70. $\int e^x\left(f\left(x\right)+f'\left(x\right)\right)dx=e^x.f\left(x\right)+c$

71. $\int e^{ax}\sin bx\,dx,\ \int e^{ax}\cos bx\,dx$, reccurring

Direct formula ,(i) $\int e^{ax}\sin bx\,dx=\frac{e^{ax}}{a^2+b^2}\left(a\sin bx-b\cos bx\right)+c$

(ii) $\int e^{ax}\cos bx\,dx=\frac{e^{ax}}{a^2+b^2}\left(a\cos bx+b\sin bx\right)+c$

72. $\int \sqrt{ax^2+bx+c}\ dx$, Completing the squares.

73. $\int (px+q)\sqrt{ax^2+bx+c}\ dx$, Write , $px+q=\lambda\frac{d}{dx}\left(ax^2+bx+c\right)+\mu$

74. Partial Fraction

75. $\int \frac{x^2+1}{x^4+\lambda x^2+1}dx$, $\int \frac{x^2-1}{x^4+\lambda x^2+1}dx$ or $\int \frac{1}{x^4+\lambda x^2+1}dx$ type, Dividing through by x^2.

76. $\int \frac{\phi(x)}{(ax+b)\sqrt{cx+d}}dx$ Put , $t^2=cx+d$

77. $\int \frac{\phi(x)}{\left(ax^2+bx+c\right)\sqrt{px+q}}dx$ Put, $t^2=px+q$

78. $\int \frac{\phi(x)}{(ax+b)\sqrt{px^2+qx+r}}dx$ Put , $ax+b=\frac{1}{t}$

79. $\int \frac{\phi(x)}{\left(ax^2+b\right)\sqrt{cx^2+d}}dx$ Put $x=\frac{1}{t}$

80. $\int \frac{f(\tan x)or1}{\sin^m x.\cos^n x}dx\ m,n\in R\ and\ m+n=even$ then divide $\sin^{m+n}x$ *or* $\cos^{m+n}x$

Inverse Trigonometry:-

1. $\sin^{-1}(\sin x)=x=\sin(\sin^{-1}x)$
2. $sin^{-1}(-x)=-\sin^{-1}x$
3. $\tan^{-1}(-x)=-\tan^{-1}x$
4. $\cos ec^{-1}(-x)=-\cos ec^{-1}x$
5. $\cos^{-1}(-x)=\pi-\cos^{-1}x$
6. $\cot^{-1}(-x)=\pi-\cot^{-1}x$
7. $\sec^{-1}(-x)=\pi-\sec^{-1}x$
8. $\sin^{-1}\left(\frac{1}{x}\right)=\cos ec^{-1}x$
9. $\cos^{-1}\left(\frac{1}{x}\right)=\sec^{-1}x$
10. $\tan^{-1}\left(\frac{1}{x}\right)=\cot^{-1}x$
11. $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$
12. $\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$
13. $\cos ec^{-1}x+\sec^{-1}x=\frac{\pi}{2}$

14. $2\tan^{-1}x=\sin^{-1}\frac{2x}{1+x^2}=\cos^{-1}\frac{1-x^2}{1+x^2}=\tan^{-1}\frac{2x}{1-x^2}$

15. $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy},xy<1$

16. $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy},xy>-1$

17. $\tan^{-x}+\tan^{-1}y=\pi+\tan^{-1}\left(\frac{x+y}{1+xy}\right)$, *for* $xy>1,x>0,y>0$

18. $\sin^{-1}x+\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}+y\sqrt{1-x^2}\right)$

19. $\sin^{-1}x-\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)$

20. $\cos^{-1}x+\cos^{-1}y=\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)$

21. $\cos^{-1}x-\cos^{-1}y=\cos^{-1}\left(xy+\sqrt{1-x^2}\sqrt{1-y^2}\right)$

22. $2\sin^{-1}x=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

23. $2\cos^{-1}x=\cos^{-1}\left(2x^2-1\right),0\le x\le 1$

24. $3\sin^{-1}x=\sin^{-1}\left(3x-4x^3\right)$

25. $3\cos^{-1}x=\cos^{-1}\left(4x^3-3x\right)$

26. $3\tan^{-1}x=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

Some Substitution Rules

For , $1-x^2$ Put $x=\sin\theta$ *or* $\cos\theta$

For, $1+x^2$, Put, $x=\tan\theta$ *or* $\cot\theta$

For, x^2-1, Put, $x=\sec\theta$ *or* $\operatorname{cosec}\theta$

For, $2x^2-1$, Put, $x=\cos\theta$

For $\sqrt{a^2-x^2}$, put $x=a\sin\theta$ *or* $a\cos\theta$

For $\sqrt{a^2+x^2}$, put $x=a\tan\theta$ *or* $a\cot\theta$

For $\sqrt{x^2-a^2}$, put $x=a\sec\theta$ *or* $a\operatorname{cosec}\theta$

For, $\frac{2x}{1+x^2}$, Put $x=\tan\theta$

For $\frac{1-x^2}{1+x^2}$, Put $x=\cos\theta$

Statistics:-

1. Mean $\left(\bar{X}\right)=\frac{\sum X_i}{n}$

$=\frac{\sum f_i X_i}{n}$

$=A+\frac{\sum f_i d_i}{n},\ where, d_i=X_i-A$

$=A+\frac{\sum f_i u_i}{n}\times h,\ where, u_i=\frac{X_i-A}{h}$

2. Combined mean:- $X_{123}=\frac{N_1\bar{X}_1+N_2\bar{X}_2+N_3X_3{}^2}{N_1+N_2+N_3}$
3. If every term is added by a, then new mean = $\bar{X}+a$
4. If every term is multiplied by a, then the new mean= $\bar{X}\cdot a$
5. Median:- $M=l+\frac{\frac{N}{2}-pcf}{f}\times h$

Where, $\frac{N}{2}$ decides the median class in the cumulative frequency column.

l=lower limit of the median class

pcf =preceding of the cumulative frequency of the median class.

f =frequency of the median class.

h=Class width

6. Mode:- $M_0=l+\frac{f_1-f_0}{2f_1-f_2-f_0}\times h$
7. Mean Deviation:-

$M.D=\frac{\sum|X_i-A|}{n}$, where, A=Mean/Median/Mode

$M.D=\frac{\sum f_i|X_i-A|}{n}$, where, A=Mean/Median/Mode

8. **Variance** $\left(\sigma^2\right)$**:-** $\sigma^2=\frac{\sum\left(x_i-\bar{X}\right)^2}{n}$

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$\sigma^2 = h^2\left[\frac{\sum u_i^2}{n} - \left(\frac{\sum u_i}{n}\right)^2\right], \text{ where } u_i = \frac{x_i - a}{h}$$

$$\sigma^2 = \frac{\sum f_i\left(x_i - \bar{X}\right)^2}{n}$$

$$\sigma^2 = \frac{\sum f_i x_i^2}{n} - \left(\frac{\sum f_i x_i}{n}\right)^2$$

$$\sigma^2 = h^2\left[\frac{\sum f_i u_i^2}{n} - \left(\frac{\sum f_i u_i}{n}\right)^2\right]$$

9. **Standard Deviation** (σ)**:-** Standard deviation is the square root of variance.

10. **Combined standard deviation:-**

$$\sigma_{123} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_3\sigma_3^2 + N_1d_1^2 + N_2d_2^2 + N_3d_3^2}{N_1 + N_2 + N_3}}$$

Where, $d_1 = \left|\bar{X}_1 - \bar{X}_{123}\right|$, $d_2 = \left|\bar{X}_2 - \bar{X}_{123}\right|$, $d_3 = \left|\bar{X}_3 - \bar{X}_{123}\right|$

11. **Coefficient of Variation**:- $C.V = \frac{\sigma}{\bar{X}} \times 100$

12. **Quartile:-** $Q_i = l + \frac{\frac{iN}{4} - pcf}{f} \times h$, $\frac{iN}{4}$ in the cumulative frequency column decides the quartile class.

(i) Inter Quartile range= $Q_3 - Q_1$

(ii) Quartile Deviation= $\frac{Q_3 - Q_1}{2}$

(iii) Coefficient of Quartile Deviation= $\frac{Q_3 - Q_1}{Q_3 + Q_1}$

(iv) Note:- The second quartile Q_2 is the median.

13. **Decile:-** $D_i = l + \frac{\frac{iN}{10} - pcf}{f} \times h$, $\frac{iN}{10}$ in the cumulative frequency column decides the Decile class.

Note:- The 5th decile *i.e.* D_5 is the median.

14. **Percentile**:- $P_i = l + \dfrac{\frac{iN}{100} - pcf}{f} \times h$, $\dfrac{iN}{100}$ in the cumulative frequency column decides the percentile class.

Note:-, $P_{25} = Q_1$, $P_{50} = Median$, $P_{75} = Q_3$

Correlation:-

1. $Cov(X,Y) = \dfrac{\sum (X - \bar{X})(Y - \bar{Y})}{n} = \dfrac{\sum xy}{n}$, (*Where* $x = X - \bar{X}$ and $y = Y - \bar{Y}$)

$= \dfrac{\sum XY}{n} - \left(\dfrac{\sum X}{n}\right)\left(\dfrac{\sum Y}{n}\right)$, (*When* $\bar{X}$ *and* $\bar{Y}$ are not integer)

$= \dfrac{\sum uv}{n} - \left(\dfrac{\sum u}{n}\right)\left(\dfrac{\sum v}{n}\right)$, (*Where* $u = X - A$ *and* $v = Y - B$)

2. *KarlPearson's Coefficient of Correlation* (ρ *or* r) :-

$\rho(X,Y) = \dfrac{Cov(X,Y)}{\sigma_X . \sigma_Y}$, (*Where* $\sigma_X = \sqrt{\dfrac{\sum (X - \bar{X})^2}{n}}$ and $\sigma_Y = \sqrt{\dfrac{\sum (Y - \bar{Y})^2}{n}}$

$= \dfrac{Cov(X,Y)}{\sqrt{Var\,X}\sqrt{Var\,Y}}$

$= \dfrac{\sum xy}{\sqrt{\sum x^2}\sqrt{\sum y^2}}$, (*Where* $x = X - \bar{X}$ and $y = Y - \bar{Y}$)

$= \dfrac{n\sum XY - \sum X \cdot \sum Y}{\sqrt{n\sum X^2 - (\sum X)^2} \cdot \sqrt{n\sum Y^2 - (\sum Y)^2}}$

$= \dfrac{n\sum UV - \sum U \cdot \sum V}{\sqrt{n\sum U^2 - (\sum U)^2} \cdot \sqrt{n\sum V^2 - (\sum V)^2}}$, ($U = \dfrac{X - A}{h}$ *and* $V = \dfrac{Y - B}{k}$)

$= \dfrac{N\sum fuv - \sum fu \cdot \sum fv}{\sqrt{N\sum fu^2 - (\sum fu)^2} \cdot \sqrt{N\sum fv^2 - (\sum fv)^2}}$ (For , bivariate)

$\rho(aX + b, cY + d) = \dfrac{a \times c}{|a||c|} \cdot \rho(X,Y)$

3. Coefficient of Determination: $(r^2) = \dfrac{Explained\ \text{variation}}{Total\ Variation}$

4. $P.E.(r) = 0.6745\frac{1-r^2}{\sqrt{n}}$

5. *Rank Correlation* :-

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} \quad = 1 - \frac{6\left[\sum d^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \cdots\right]}{n(n^2-1)}$$

6. *Regression Equation of X on Y is* :- $X - \bar{X} = b_{XY}(Y - \bar{Y})$

Regression Equation of Y on X is :- $Y - \bar{Y} = b_{YX}(X - \bar{X})$

$$b_{XY} = \frac{n\sum XY - (\sum X)\cdot(\sum Y)}{n\sum Y^2 - (\sum Y)^2} \quad , \quad b_{YX} = \frac{n\sum XY - (\sum X)\cdot(\sum Y)}{n\sum X^2 - (\sum X)^2}$$

$$= \frac{n\sum UV - (\sum U)(\sum V)}{n\sum V^2 - (\sum V)^2} \qquad = \frac{n\sum UV - (\sum U)(\sum V)}{n\sum U^2 - (\sum U)^2} \quad (U = X - A, V = X - B)$$

$$= \frac{n\sum fuv - (\sum fu)(\sum fv)}{n\sum fv^2 - (\sum fv)^2} \times \frac{h}{k}, \quad = \frac{n\sum fuv - (\sum fu)(\sum fv)}{n\sum fu^2 - (\sum fu)^2} \times \frac{h}{k} \left(u = \frac{X - A}{h},\right)$$

$$= \frac{Cov(X,Y)}{\sigma_Y^2} \qquad = \frac{Cov(X,Y)}{\sigma_X^2}$$

$$= \frac{\sum xy}{\sum y^2} \qquad = \frac{\sum xy}{\sum x^2} \qquad (\textit{Where } x = X - \bar{X} \text{ and } y = Y - \bar{Y})$$

$$= r\frac{\sigma_x}{\sigma_y} \qquad = r\frac{\sigma_y}{\sigma_x}$$

If $X = a + bY$, then $b = b_{XY}$ & If $Y = a + bX$, then $b = b_{YX}$